Marketing Strategy Using Markovian Model on Long Term Steady State Market Share

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ABSTRACT

In this paper, there is the attempt of examining the long run steady state market shares of a variety of soap in Sambalpur, Odisha by applying the Markovian theory. The matrix consists of possibilities of transition of consumers who is switching different brand of bathing soap. The Markovian approach gives an insight of the reason of customer switching and try to identify the potential completion of a brand in the market and predicts the future condition of gaining and losing the customers of a particular brand. All the calculations done by primary data and using this data probability matrix has been designed.

Keywords: - Brand, Switching, Bath soap, share, Markov chain.

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1. Introduction

In marketing, an entrepreneurial activity (An off, 1987), the practice of creating a name, symbol or design that identifies and differentiates a product from other product is known as branding. Basically, brand switching is the loss of own loyal customer to other brands and winning the loyal customers of other brands.

Out of the commonly used brands switching models of marketing, one extremely popular model is the Markovian brand switching model. A.A. Markov, Russian mathematician invented Markov chain in the early 1900's. It is a method based on mathematical model with the consideration of a random phenomenon. This method was constructed by imposing a crucial additional assumption on discrete time stochastic process. The concerned random phenomenon changes with time. Though the model was invented in early 1900's but the major applications of the model started in the second half of nineteenth century. The applications were principally done on brand loyalty studies.

Sometimes consumers wish to try other brands for the same products but they are dissatisfied in choosing that as it does not fulfill their expectation. It is not that the preference of choosing the particular brands and products depends on the quality and durability of the product but also on the preference of consumer's conditions and situations. In other words, the consumer's reaction towards the product can be affected by the cost and price of that particular product. Price sensitivity or price elasticity means how much the sales of the product can be influenced by the price. The consumers are now easily choosing the other brand because of the price sensitivity.

The company needs to be careful enough especially in maintaining the price sensitivity in bath soap and chocolates as consumers are quickly changing their minds for another brand. Even though a consumer has a habit of preferring a particular brand yet if he notice and realize that the price, he paid for the brand is not reasonable then at the very moment the brand will lose its customers.

Many people consider that when the product's Price is decreased the consumer will have a craze for the product. Getting decrease in the price is not the solution to attract consumers. The probability is that it may create confusion and doubt as the customers are given with different choices especially in choosing price for the same product. Then they often choose to the one that they are accustomed with for a long time. When they are given more option in choosing the product with different price, it become hard to determine for them that which one is to be chosen. The consumer will never go for the unknown products and wants to choose the familiar product when there is confusion in his mind. Thus, consumers have more acceptance towards the brand and has emotional attachment to it than with the lower price. Brand has a vital role in maintaining consumer's certainty.

Brand loyalty is an area of concern for marketers in India as a lot of brand switching is observed across many product categories. It was found in the global survey conducted by Accenture that "Indian consumers are much more inclined to switch brands and companies than in other nations because of poor service, bringing on a potential income loss of \$331 billion for big firms". In India Global Consumer Pulse Survey conducted by Accenture discovered that "88% of respondents switched or not loyal to companies because of poor service". This score is beyond the global average of 64%. Hence it is an alarming situation to the India marketer to focus on the issue of brand switching.

Product: According to Philip Kotler, "Product is anything that can be offered to someone to satisfy a need or want".

Brand : According to American marketing association, "a brand is a name, term, sign, symbol or design or a combination of them, intended to identify the goods and services of one seller or group of sellers and to differentiate then from those of competitors".

Market Share: Market share is explained as part of a market that run by a company, or a company's sales percentage of the total sales compared to the outstanding competitors with a particular time and place.

However, to the best of our information, no researches have been done on the application of Markov analysis on the brand switching related to bath soap.

1.2. Research Methodology

The total number of 242 sample of customers who are using bath soap has been taken from the students of Gangadhar Meher University, Sambalpur, Odisha to conduct the current research. The purposive sampling techniques have been applied for the research. All 242 customers are given with a set of questionnaires by Google form through online to collect the primary data directly from them. So, the data that has been collected for this research is purely based on primary data or first-hand data. The questionnaire is intended to get information from the students. The brand of bath soap that they are presently using as well as they use to use in past. The data that is collected from the student's response are subjected to the Markov chain application and steady state equation. And the study predicts the possibility of buying same brand of products of bath soap or possibility of changing one brand to other brand of bath soap. This analysis and calculations are exclusively done in R-programming and excel.

2. Markov Analysis

Markov chain refers to a specific category of discrete time stochastic process. If X_t denotes the value taken by a system property at time t (t can be any of the discrete point in time that may be labelled as 0, 1, 2, 3, ...), then in most of the situations, it is not possible to accurately know the value of X_t until time t comes. Hence, X_t is treated as a random variable. An exhibition of the relation between X_{ts} (t=0,1,2,3,...) is known as a discrete-time stochastic process.

Let there be 'k' (finite) number of states numbered as $1, 2 \dots k$. Let us also assume that at any time t (t can take any values like $0, 1, 2, 3 \dots$) a discrete-time stochastic process is in any one of the above-mentioned states. Then in order to qualify as a Markov chain, it has to satisfy the following condition:

 $P(X_{t+1} = u_{t+1} | X_t = u_t, X_{t-1} = u_{t-1} \dots X_1 = u_1, X_0 = u_0) = P(X_{t+1} = u_{t+1} | X_t = u_t)$ Further

 $P(X_{t+1} = v | X_t = u) = p_{uv}$ where p_{uv} is the probability of transition of a system property from state u to state v in one time period. It is not dependent on the absolute value of t (but depends on the time gap), for stationary Markov chains.

The $(k \times k)$ matrix one – step transition probability is given by

$$P = \begin{bmatrix} p_{11} p_{12} & \cdots & p_{1k} \\ p_{21} p_{22} & \cdots & p_{2k} \\ \vdots & \vdots & \vdots \\ p_{k1} p_{k2} & \cdots & p_{kk} \end{bmatrix}, \sum_{\nu=1}^{k} p_{u\nu} = 1, \forall u$$

The probability of transition of a system property a system property from state v in n time periods for a stationary Markov chain (which is in state u at time), is given by

$$P(X_{t+n} = v | X_t = u) = P(X_n = v | X_0 = u) = P_{uv}^{(n)},$$

where $P_{uv}^{(n)}$ is known as n-step transition probability and is independent of t.

It is known that when n takes integer values greater than 1, then $P_{uv}^{(n)}$ = the value occupying the (u, v) th position/cell of the matrix P^n .

Now let us discuss on the concept of steady-state probabilities. If we consider a *k*-state Markov chain, there has to be a vector $\Pi = [\Pi_1 \Pi_2 \Pi_3 \dots \Pi_k]$, such that

$$\lim_{m \to \infty} P^{m} = \begin{bmatrix} \Pi_{1} & \Pi_{2} & \dots & \Pi_{k} \\ \Pi_{1} & \Pi_{2} & \dots & \Pi_{k} \\ \vdots & \vdots & \ddots & \ddots \\ \vdots & \vdots & \ddots & \ddots \\ \Pi_{1} & \Pi_{2} & \dots & \Pi_{k} \end{bmatrix} \text{ i.e. } \lim_{m \to \infty} P_{uv}^{(m)} = \Pi_{v}$$

The vector $\Pi = [\Pi_1 \Pi_2 \Pi_3 \dots \Pi_k]$ is known as the steady state distribution or equilibrium distribution and $\Pi_1 \Pi_2 \Pi_3 \dots \Pi_k$ are the steady state probabilities. Now, at the steady state probabilities, the following equations are to be solved:

$$\Pi = \Pi P \tag{1}$$

(2)

and

$\sum_{\nu=1}^{k} \Pi_{\nu} = 1$

2.1 Analysis and findings

The current market share of each of the bath soap brand and the transition probability matrix is calculated. We have attempted to examine the long run steady state market shares of a variety of soap in Sambalpur, Odisha by applying the Markovian theory and also predict future market share of bath soap.

2.2 Current Market Shares and Previously Market Shares

According to paper "Markov chain model in measure brand switching of bath soap"(table 2.5) by Jena SP et al., (2022) the responses obtained, regarding the bath soap brands currently and previously used by respondents of our sample, the present market shares of various bath soap brands in the G.M University students are estimated to as follow (as depicted in fig. 3.1): Dove-9%, Dettol-14%, Lifebuoy-11%, Pears-3%, Santoor-3%, Mysore sandal-9%, Medimix-3%, Cinthol-11%, Patanjali-8%, Himalay-2%, Vivel-4%, Liril-1%, Wildstone-5%, Specify-2%, Lux-7%, Other-8% and the previous market shares of various bath soap brands in the G.M University students are estimated to as follow (as depicted in fig. 3.2): Dove-10%, Dettol-13%, Lifebuoy-15%, Pears-6%, Santoor-6%, Mysore sandal-4%, Medimix-2%, Cinthol-10%, Patanjali-9%, Himalay-2%, Vivel-3%, Liril-2%, Wildstone-2%, Specify-1%, Lux-10%, Other-5%.







Fig.2.2

From the responses obtained from each of the 242 respondents regarding the bath soap brands that they expected to purchase in their next purchase, the following data be made up of the frequency of respondent switching their brands and also the frequency of G.M University students retaining their old brands obtained (table3.1)

Table	3.1
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Brand	dove	dettol	life bouy	pears	santoor	mysore	medimix	cinthol	patanjali	himalaya	vivel	liril	wild stone	specify	lux	other	total
dove	7	3	1	3	0	1	0	1	0	0	0	1	0	0	3	3	23
dettol	2	12	5	1	0	1	0	6	2	0	0	0	1	1	1	0	32
life bouy	3	7	13	0	1	2	1	2	2	0	2	1	1	0	1	. 1	37
pears	1	0	0	2	0	3	2	1	1	1	0	0	0	0	3	0	14
santoor	1	1	1	0	2	1	0	2	0	1	2	0	0	1	1	0	13
mysore	0	1	0	0	0	6	0	0	1	0	0	0	2	0	0	0	10
medimix	0	0	0	0	0	0	2	1	0	0	0	0	1	0	0	1	5
cinthol	0	1	1	0	1	3	0	9	4	0	0	0	4	0	0	2	25
patanjali	2	1	0	0	0	3	0	0	10	0	1	0	1	0	1	3	22
himalaya	1	0	0	0	0	0	0	0	0	3	0	0	1	0	0	0	5
vivel	1	2	1	0	0	0	0	0	0	0	3	0	0	0	0	0	7
liril	0	0	0	0	0	1	1	1	0	0	0	0	0	0	1	0	4
wild stone	1	0	0	0	0	0	0	2	0	0	0	0	1	0	0	1	5
specify	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	2
lux	2	4	4	2	2	0	0	2	0	0	1	0	0	1	6	1	25
other	0	2	1	0	0	0	1	0	0	0	0	1	0	1	1	6	13

Taking each row in the above table, on dividing each cell frequency of the corresponding row, the matrix of transition probabilities (considering one time gap / one step) is obtained and displayed below

The above transition probability matrix (P) convert into three decimal place transition

probability matrix shows below table 3.2

Brand	Dove	Dettol	Lifebuoy	Pears	Santoor	Mysore	Medimix	Cinthol	Patanjali	Himalaya	Vivel	Liril	Wild stone	Specify	Lux	Other
Dove	0.304	0.130	0.043	0.130	0.000	0.043	0.000	0.043	0.000	0.000	0.000	0.043	0.000	0.000	0.130	0.130
Dettol	0.063	0.375	0.156	0.031	0.000	0.031	0.000	0.188	0.063	0.000	0.000	0.000	0.031	0.031	0.031	0.000
Life buoy	0.081	0.189	0.351	0.000	0.027	0.054	0.027	0.054	0.054	0.000	0.054	0.027	0.027	0.000	0.027	0.027
Pears	0.071	0.000	0.000	0.143	0.000	0.214	0.143	0.071	0.071	0.071	0.000	0.000	0.000	0.000	0.214	0.000
Santoor	0.077	0.077	0.077	0.000	0.154	0.077	0.000	0.154	0.000	0.077	0.154	0.000	0.000	0.077	0.077	0.000
Mysore	0.000	0.100	0.000	0.000	0.000	0.600	0.000	0.000	0.100	0.000	0.000	0.000	0.200	0.000	0.000	0.000
Medimix	0.000	0.000	0.000	0.000	0.000	0.000	0.400	0.200	0.000	0.000	0.000	0.000	0.200	0.000	0.000	0.200
Cinthol	0.000	0.040	0.040	0.000	0.040	0.120	0.000	0.360	0.160	0.000	0.000	0.000	0.160	0.000	0.000	0.080
Patanjali	0.091	0.045	0.000	0.000	0.000	0.136	0.000	0.000	0.455	0.000	0.045	0.000	0.045	0.000	0.045	0.136
Himalaya	0.200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.600	0.000	0.000	0.200	0.000	0.000	0.000
Vivel	0.143	0.286	0.143	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.429	0.000	0.000	0.000	0.000	0.000
Liril	0.000	0.000	0.000	0.000	0.000	0.250	0.250	0.250	0.000	0.000	0.000	0.000	0.000	0.000	0.250	0.000
Wild stone	0.200	0.000	0.000	0.000	0.000	0.000	0.000	0.400	0.000	0.000	0.000	0.000	0.200	0.000	0.000	0.200
Specify	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.500	0.000	0.500
Lux	0.080	0.160	0.160	0.080	0.080	0.000	0.000	0.080	0.000	0.000	0.040	0.000	0.000	0.040	0.240	0.040
Other	0.000	0.154	0.077	0.000	0.000	0.000	0.077	0.000	0.000	0.000	0.000	0.077	0.000	0.077	0.077	0.462

Using the transition probability matrix (P) and steady-state equations [equation (3),(4),(5),(6),(7),(8),(9),(10),(11),(12),(13),(14),(15),(16),(17),(18),(19)] have been established with help of equations (1) and (2). It is done on the from the table 3.2 gives out some important facts. Dove has 30% of its customers as brand loyal customers. However, it has to be careful with Dettol, Pears, Lux and Other as its main four contestants, since it is taking away around 13% of its customers. 38% customers of Dettol occur to be brand loyal. 35%, 14%, 15%, 60%, 40%, 36%, 46%, 60%, 43%, 20%, 50%, 24%, customers of Lifebuoy, Pears, Santoor, Mysore sandal, Medimix, Cinthol, Patanjali, Himalaya, Vivel, Wildstone, specify, Lux appear to be brand loyal respectively. Liril has no loyal customer. That is 0% of customers of liril appear to be brand loyal. The above given table shows that the maximum brands are connected to each other. These involve that all the promotional and advertising strategies of brands should be directed against each other brands and vice versa. The table further reveals that the bath soap brand with highest percentage of brand loyal customers are Mysore sandal and Himalaya (60%). Another salient feature for Mysore sandal and Himalaya. Their customers are not at all interested in purchasing other brand in next purchase.

Basic of the usual Markovian assumption that during the period under deliberation, none of the bath soap brands have employed any radical change in their marketing strategies in differentiation to the strategies they are presently using.

Now, equation (1) can be written as,

$$\begin{bmatrix} \Pi_1 \Pi_2 \Pi_3 \Pi_4 \Pi_5 \Pi_6 \Pi_7 \Pi_8 \Pi_9 \Pi_{10} \Pi_{11} \Pi_{12} \Pi_{13} \Pi_{14} \Pi_{15} \Pi_{16} \end{bmatrix}^* P = \begin{bmatrix} \Pi_1 \Pi_2 \Pi_3 \Pi_4 \Pi_5 \Pi_6 \Pi_7 \Pi_8 \Pi_9 \Pi_{10} \Pi_{11} \Pi_{12} \Pi_{13} \Pi_{14} \Pi_{15} \Pi_{16} \end{bmatrix}$$

Or
$$\begin{bmatrix} \frac{7}{12} \Pi_1 + \frac{1}{12} \Pi_2 + \frac{3}{27} \Pi_3 + \frac{1}{14} \Pi_4 + \frac{1}{12} \Pi_5 + \frac{1}{14} \Pi_9 + \frac{1}{5} \Pi_{10} + \frac{1}{5} \Pi_{11} + \frac{1}{5} \Pi_{13} + \frac{2}{27} \Pi_{15} = \Pi_1 \quad (3)$$

$$\frac{3}{23}\Pi_{1} + \frac{3}{8}\Pi_{2} + \frac{7}{37}\Pi_{3} + \frac{1}{13}\Pi_{5} + \frac{1}{10}\Pi_{6} + \frac{1}{25}\Pi_{8} + \frac{1}{22}\Pi_{9} + \frac{2}{7}\Pi_{11} + \frac{4}{25}\Pi_{15} + \frac{2}{13}\Pi_{16} = \Pi_{2}$$
(4)

$$\frac{1}{23}\Pi_1 + \frac{5}{32}\Pi_2 + \frac{13}{37}\Pi_3 + \frac{1}{13}\Pi_5 + \frac{1}{25}\Pi_8 + \frac{1}{7}\Pi_{11} + \frac{4}{25}\Pi_{15} + \frac{1}{13}\Pi_{16} = \Pi_3$$
(5)

$$\frac{1}{23}\Pi_1 + \frac{1}{32}\Pi_2 + \frac{1}{7}\Pi_4 + \frac{1}{25}\Pi_{15} = \Pi_4$$
(6)
$$\frac{1}{12}\Pi_3 + \frac{2}{12}\Pi_5 + \frac{1}{12}\Pi_8 + \frac{2}{12}\Pi_{15} = \Pi_5$$
(7)

$$\frac{1}{23}\Pi_{1} + \frac{1}{32}\Pi_{2} + \frac{2}{37}\Pi_{3} + \frac{3}{14}\Pi_{4} + \frac{1}{13}\Pi_{5} + \frac{3}{5}\Pi_{6} + \frac{3}{25}\Pi_{8} + \frac{3}{22}\Pi_{9} + \frac{1}{4}\Pi_{12} = \Pi_{6}$$
(8)

$$\frac{1}{37}\Pi_3 + \frac{1}{7}\Pi_4 + \frac{2}{5}\Pi_7 + \frac{1}{4}\Pi_{12} + \frac{1}{13}\Pi_{16} = \Pi_7 \tag{9}$$

$$\frac{1}{23}\Pi_1 + \frac{3}{16}\Pi_2 + \frac{2}{37}\Pi_3 + \frac{1}{14}\Pi_4 + \frac{2}{13}\Pi_5 + \frac{1}{5}\Pi_7 + \frac{9}{25}\Pi_8 + \frac{1}{4}\Pi_{12} + \frac{2}{5}\Pi_{13} + \frac{2}{25}\Pi_{15} = \Pi_8 \tag{10}$$

$$\frac{1}{16}\Pi_2 + \frac{2}{37}\Pi_3 + \frac{1}{14}\Pi_4 + \frac{1}{10}\Pi_6 + \frac{1}{25}\Pi_8 + \frac{3}{11}\Pi_9 = \Pi_9$$
(11)

$$\frac{1}{14}\Pi_4 + \frac{1}{13}\Pi_5 + \frac{1}{5}\Pi_{10} = \Pi_{10} \tag{12}$$

$$\frac{2}{37}\Pi_3 + \frac{2}{13}\Pi_5 + \frac{1}{22}\Pi_9 + \frac{3}{7}\Pi_{11} + \frac{1}{15}\Pi_{15} = \Pi_{11}$$
(13)

$$\frac{1}{23}\Pi_1 + \frac{1}{37}\Pi_3 + \frac{1}{13}\Pi_{16} = \Pi_{12} \tag{14}$$

$$\frac{1}{32}\Pi_2 + \frac{1}{37}\Pi_3 + \frac{1}{5}\Pi_6 + \frac{1}{5}\Pi_7 + \frac{4}{25}\Pi_8 + \frac{1}{22}\Pi_9 + \frac{1}{5}\Pi_{10} + \frac{1}{5}\Pi_{13} = \Pi_{13}$$
(15)

$$\frac{1}{32}\Pi_2 + \frac{1}{32}\Pi_5 + \frac{1}{2}\Pi_{14} + \frac{1}{25}\Pi_{15} + \frac{1}{13}\Pi_{16} = \Pi_{14}$$
(16)

$$\frac{3}{23}\Pi_1 + \frac{1}{32}\Pi_2 + \frac{1}{37}\Pi_3 + \frac{3}{14}\Pi_4 + \frac{1}{13}\Pi_5 + \frac{1}{22}\Pi_9 + \frac{1}{4}\Pi_{12} + \frac{6}{25}\Pi_{15} + \frac{1}{13}\Pi_{16} = \Pi_{15}$$
(17)

$$\frac{3}{23}\Pi_1 + \frac{1}{13}\Pi_3 + \frac{1}{5}\Pi_7 + \frac{2}{25}\Pi_8 + \frac{3}{22}\Pi_9 + \frac{1}{5}\Pi_{13} + \frac{1}{2}\Pi_{14} + \frac{2}{25}\Pi_{15} + \frac{6}{13}\Pi_{16} = \Pi_{16}$$
(18)
And equation (2) can be expressed as,

$$\Pi_{1} + \Pi_{2} + \Pi_{3} + \Pi_{4} + \Pi_{5} + \Pi_{6} + \Pi_{7} + \Pi_{8} + \Pi_{9} + \Pi_{10} + \Pi_{11} + \Pi_{12} + \Pi_{13} + \Pi_{14} + \Pi_{15} + \Pi_{16} = 1$$
(19)

Solving the above set of equations by using 'R' software, the following solution has been obtained

These values hand out the long run predicted market share of each brand.

 $\pi_1 = 0.068345118$ (Dove);

 $\pi_2 = 0.127780837$ (Dettol);

 $\pi_3 = 0.077704026$ (Lifebuoy);

 $\pi_4 = 0.019766033$ (Pears);

 $\pi_5 = 0.013092861$ (Santoor);

 $\pi_6 = 0.115902045$ (Mysore sandal);

 $\pi_7 = 0.032091304$ (Medimix);

 $\pi_8 = 0.123597104$ (Cinthol);

 $\pi_9 = 0.082434221$ (Patanjali);

 $\pi_{10} = 0.006047507$ (Himalaya);

 $\pi_{11} = 0.020962909$ (Vivel);

 $\pi_{12} = 0.015522072$ (Liril);

 $\pi_{13} = 0.075529972$ (Wildstone);

 $\pi_{14} = 0.034936044$ (Specify);

 $\pi_{15} = 0.050432272$ (Lux);

 $\pi_{16} = 0.135855675$ (Other).

Thus, the market shares of several bath soap brands in the long run will be as follows (Fig.3.3):

$$\pi_1 = 6.83\%$$
(Dove);
 $\pi_2 = 12.78\%$ (Dettol);

 $\pi_3 = 7.77\%$ (Lifebuoy);

 $\pi_4 = 1.98\%$ (Pears);

 $\pi_5 = 1.31\%$ (Santoor);

 $\pi_6 = 11.59\%$ (Mysoresandal);

$$\pi_7 = 3.21\%$$
(Medimix);

$$\pi_8 = 12.36\%$$
(Cinthol);

Journal of Statistics, Optimization and Data Science Vol. 1 No: 1 (June 2023); pp 1-11 $\pi_9 = 8.24\%$ (Patanjali); $\pi_{10} = 0.60\%$ (Himalaya); $\pi_{11} = 2.1\%$ (Vivel); $\pi_{12} = 1.55\%$ (Liril); $\pi_{13} = 7.55\%$ (Wildstone); $\pi_{14} = 3.49\%$ (Specify); $\pi_{15} = 5.04\%$ (Lux); $\pi_{16} = 13.59\%$ (Other).



From the above figure, it is very much clear that the current market share and future market share or long term market share are differently presented. The future market share or long run market share states that 12.78 %(Dove), 12.36% (Cinthol) and 11.59% (Mysore sandal) will be the leading brand in the market in coming future. However, Dettol will replace lifebuoy as the main brands.

3. Conclusion

Here, in this paper the researcher attempts to examine the long run steady state market shares of a variety of soap in Sambalpur, Odisha by applying the Markovian theory. The matrix consist of possibilities of transition of consumers who is switching different brand of bathing soap. The Markovian approaches gives an insight of the reason of customer switching and try to identify the potential completion of a brand in the market and predicts the future condition of gaining and losing the customers of a particular brand.

The similar analysis should be applied by taking different products for future study. Moreover, by considering the transition probability, one may set different problem to be imprecise and solve the same using stochastic or fuzzy or interval arithmetic approach.

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Journal of Statistics, Optimization and Data Science Vol. 1 No: 1 (June 2023); pp 1-11 **R**-Programming coding P=matrix(c(7/23,3/23,1/23,3/23,0,1/23,0,1/23,0,0,0,1/23,0,0,3/23,3/23,1/16,3/8,5/32,1/32,0,1/32, 0,3/16,1/16,0,0,0,1/32,1/32,1/32,0,3/37,7/37,13/37,0,1/37,2/37,1/37,2/37,2/37,0,2/37,1/37,1/37,0 ,1/37,1/37,1/14,0,0,1/7,0,3/14,1/7,1/14,1/14,1/14,0,0,0,0,3/14,0,1/13,1/13,0,2/13,1/13,0,2/1 1/5,0,0,1/5,0,1/25,1/25,0,1/25,3/25,0,9/25,4/25,0,0,0,4/25,0,0,2/25,1/11,1/22,0,0,0,3/22,0,0,5/11, 0,0,0,0,1/2,0,1/2,2/25,4/25,4/25,2/25,2/25,0,0,2/25,0,0,1/25,0,0,1/25,6/25,1/25,0,2/13,1/13,0,0,0, 1/13,0,0,0,0,1/13,0,1/13,1/13,6/13),nrow=16,ncol=16,byrow=T) print(P) 0,0,0,0,1),nrow=16,ncol=16,byrow=T) print(i) c=t(P-i)print(c) d=det(c)print(d) print(e) f=cbind(c,e) print(f) g=det(c)print(g) h=rbind(c,c(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1))) print(h) u=cbind(h,e) print(u) k=qr.solve(h,e) print(k)